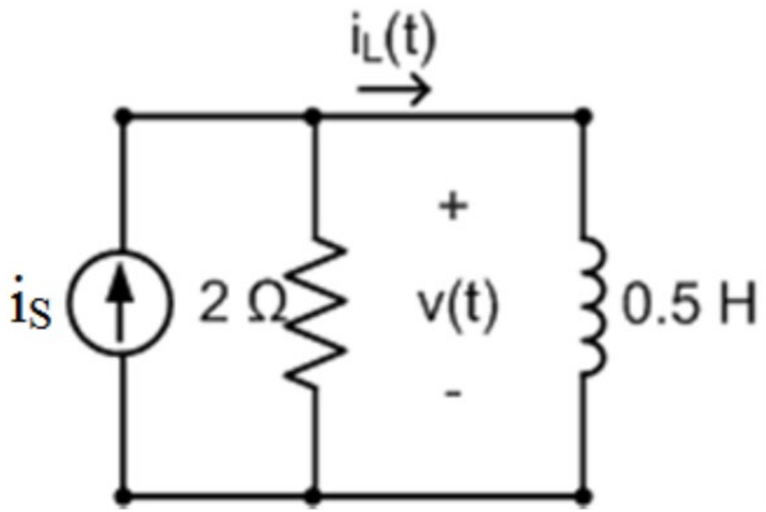
Review for quiz 2

Transfer functions

- Ratio of output quantity (V or I) to input quantity (V or I source) in s-domain: H(s)
- If input is G(s), then output is F(s)=H(s)G(s) (multiplication)
- ILT of H(s) is impulse response h(t).
- If the input is g(t), then the output is f(t)=h(t)*g(t) (convolution).

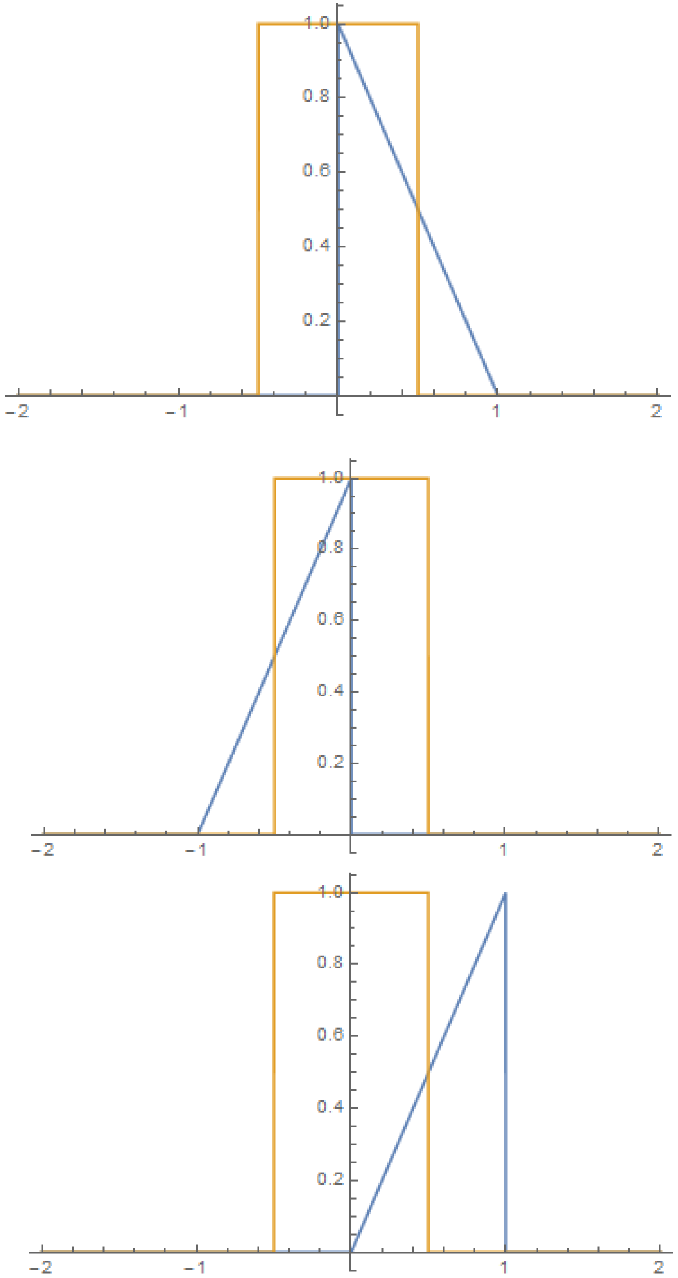
$I_L(s)/I_S(s)=4/(s+4)$
$i_S(t)=\delta(t)$, $i_L(t)=$?

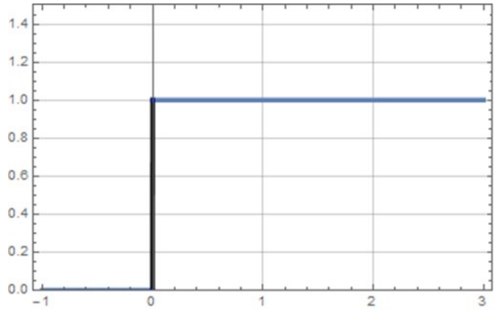


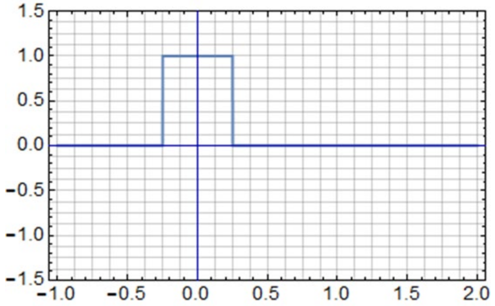
a. 12 A
b. 6 A
c. $8\,e^{-4t}$ A
d. e^{-4t} A
e. $4\,e^{-4t}$ A

Convolution

- Given g(t) and h(t)
- f=g*h= $\int_{-\infty}^{\infty} g(\lambda)\, h(t-\lambda)\, d\lambda$
- Properties
  - Commutative
  - Associative
  - Distributive



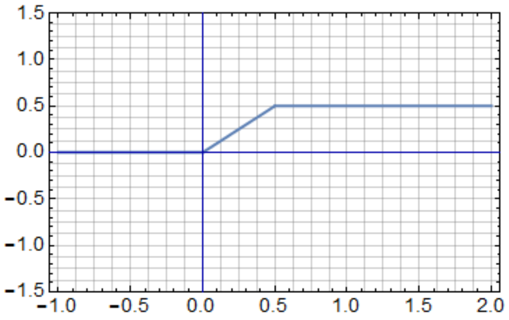


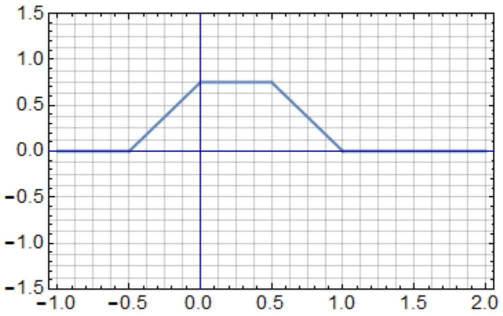


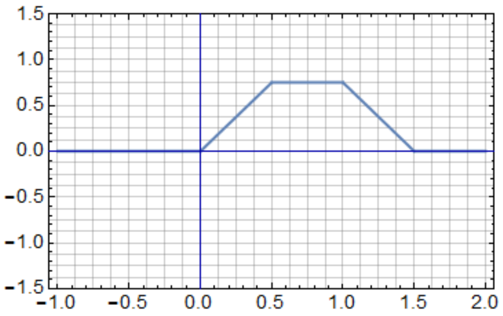
A.

B.

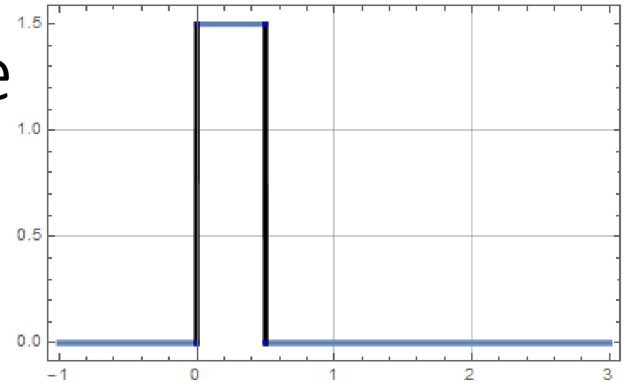
C.

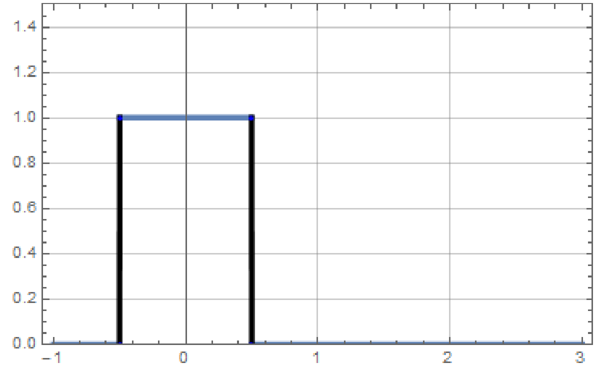






Convolve





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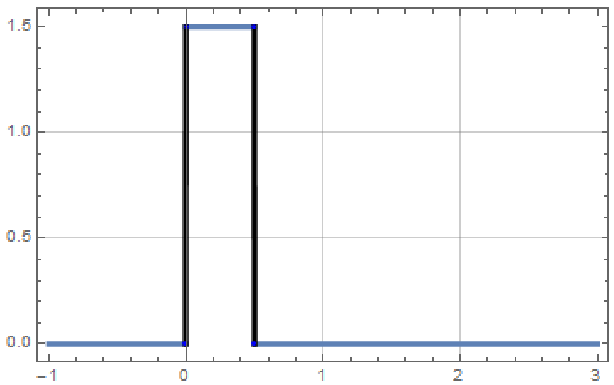
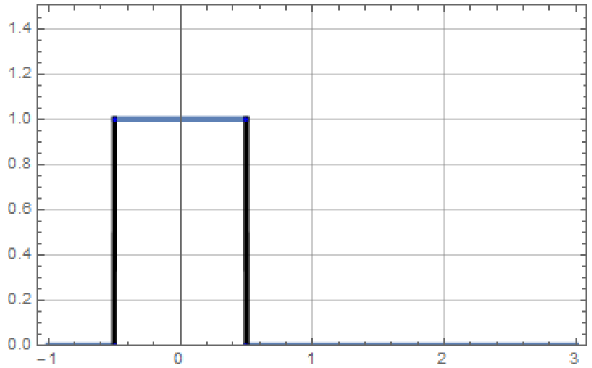
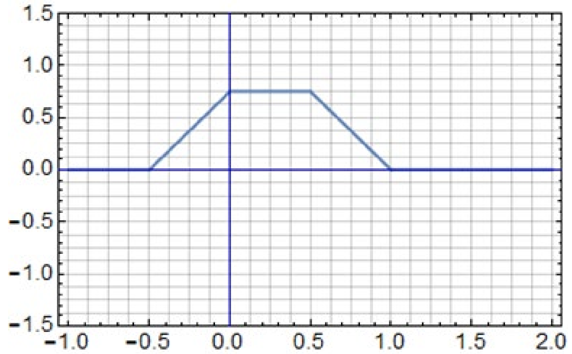
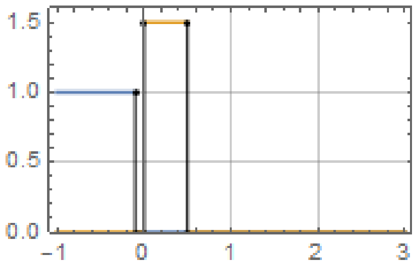
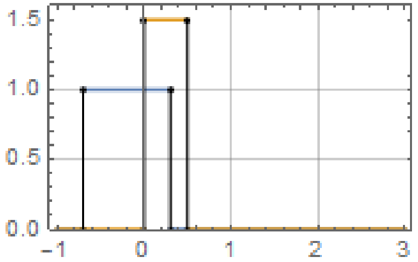
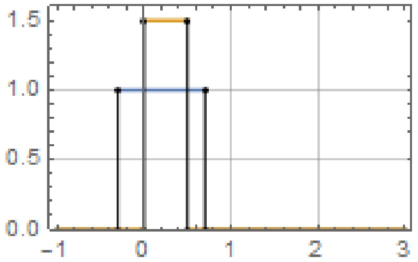
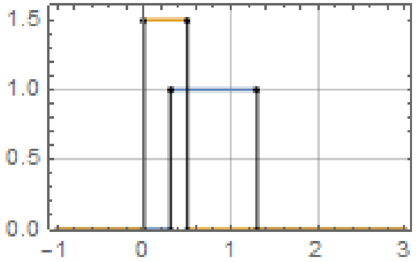
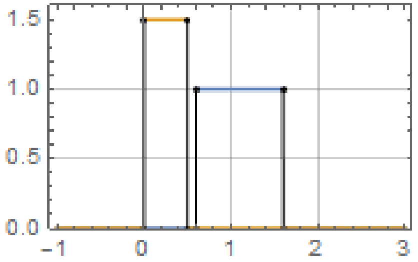
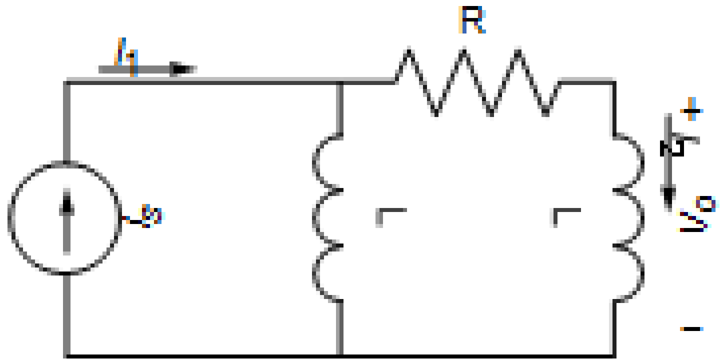




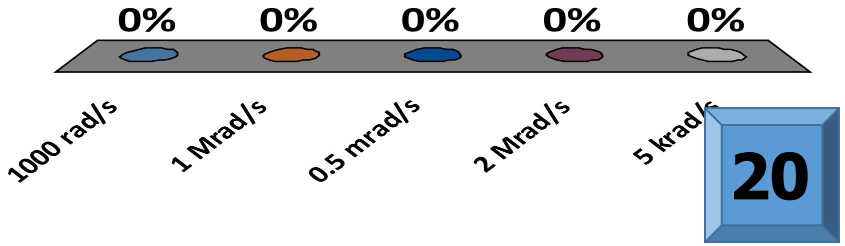


$t \leqslant -1/2$	0	
$-1/2 \leqslant t \leqslant 0$	$\int_0^{t+1/2} 1.5 \times 1 \, d\lambda = \frac{3}{2} t + \frac{3}{4}$	
$0 \leqslant t \leqslant 1/2$	$\int_0^{1/2} 1.5 \times 1 \, d\lambda = \frac{3}{4}$	
$1/2 \leqslant t \leqslant 1$	$\int_{t-1/2}^{1/2} 1.5 \times 1 \, d\lambda = -\frac{3}{2} t + \frac{3}{2}$	
$1 \leqslant t$	0	

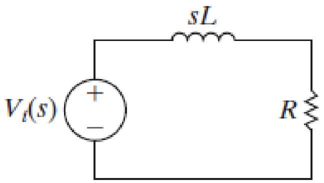
$H(j\omega)=I_2/I_s$, R= 1 kΩ, L=0.5 mH. The half power angular cut-off frequency ω_c is

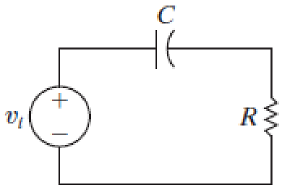


A. 1000 rad/s
B. 1 Mrad/s
C. 0.5 mrad/s
D. 2 Mrad/s
E. 5 krad/s



Frequency responses: low pass, band pass, high pass, band stop
first order: high pass or low pass





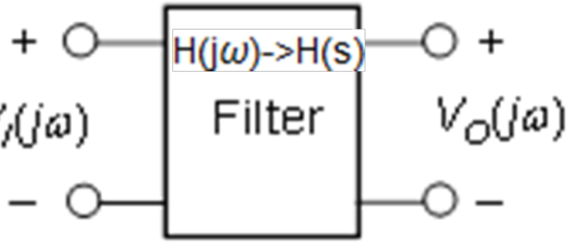
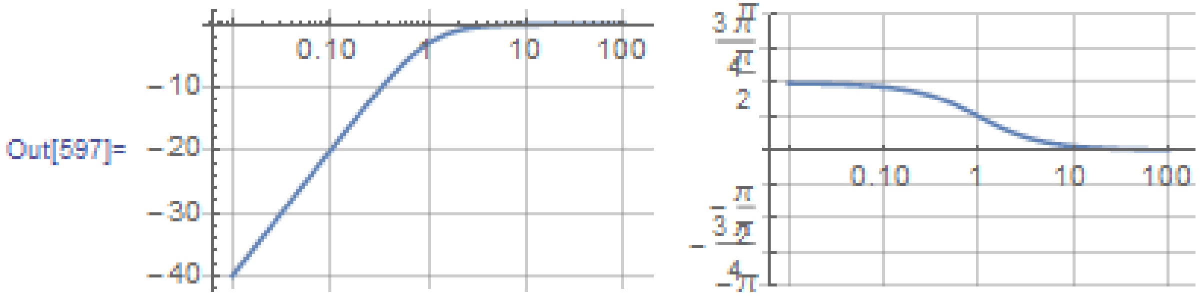


Figure 14.1.1

$$H(j\omega) = \frac{R}{R+1/j\omega C} = \frac{j\omega RC}{j\omega RC+1} = \frac{j\omega/\omega_c}{j\omega/\omega_c+1}$$

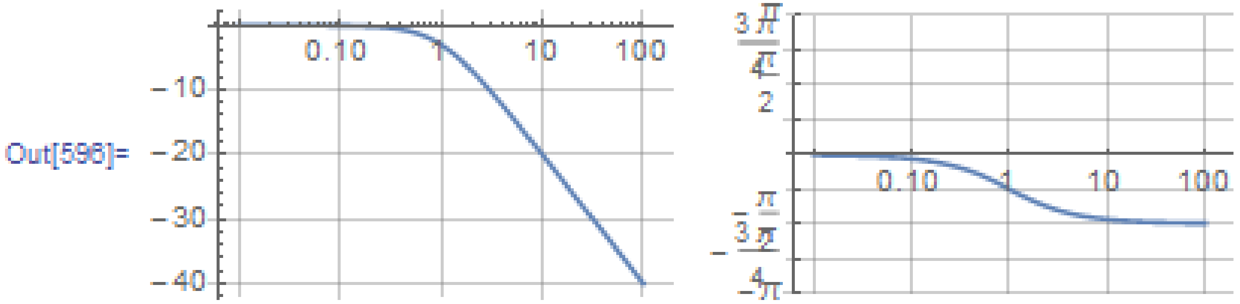
$$H(j\omega) = \frac{j\omega L}{R+j\omega L} = \frac{j\omega L/R}{j\omega L/R+1} = \frac{j\omega/\omega_c}{j\omega/\omega_c+1}$$



ω_c=1/τ=1/RC or 1/GL=R/L

$$H(j\omega) = \frac{1/j\omega C}{R+1/j\omega C} = \frac{1}{j\omega RC+1} = \frac{1}{j\omega/\omega_c+1}$$

$$H(j\omega) = \frac{R}{R+j\omega L} = \frac{1}{j\omega L/R+1} = \frac{1}{j\omega/\omega_c+1}$$



2nd order frequency responses summary

$j\omega \to s$, $B = \omega_2 - \omega_1$, $Q = \frac{\omega_0}{B}$, $\omega_1 \omega_2 = \omega_0^2$

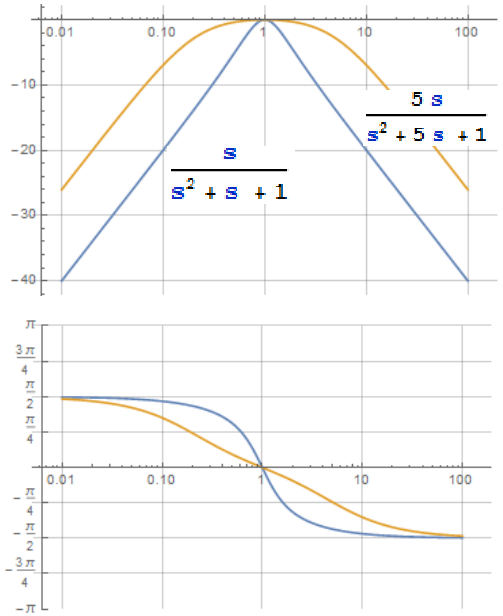
$K\frac{s^2}{s^2+Bs+\omega_0^2}$ highpass

$K\frac{Bs}{s^2+Bs+\omega_0^2}$ bandpass

$K\frac{\omega_0^2}{s^2+Bs+\omega_0^2}$ lowpass

$K\frac{s^2+\omega_0^2}{s^2+Bs+\omega_0^2}$ bandstop

$K\frac{s^2-Bs+\omega_0^2}{s^2+Bs+\omega_0^2}$ allpass



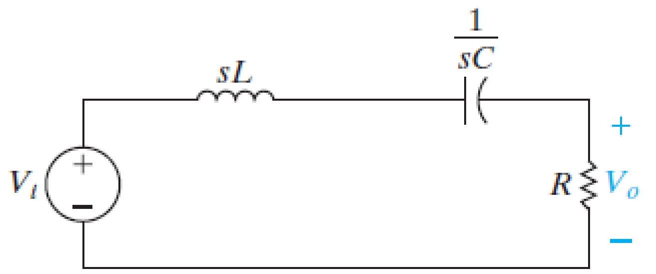
ω_0 is resonance frequency.

B is bandwidth.

Q is quality factor.

ω_1, ω_2 are half power frequencies.

$$\omega_{1,2} = \mp\frac{B}{2} + \sqrt{\left(\frac{B}{2}\right)^2 + \omega_0^2}$$



$$2\text{ nd order}: \frac{R}{R + j\omega L + \frac{1}{j\omega C}} = \frac{\frac{R}{L}s}{s^2 + \frac{R}{L}s + \frac{1}{LC}}$$

Series RCL

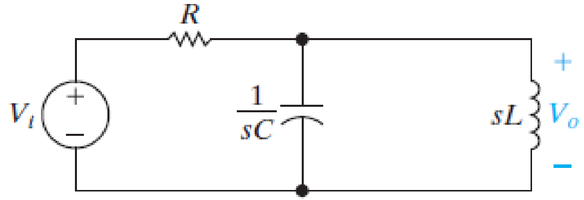
$\omega_0^2 = \frac{1}{LC}$, $B = \frac{R}{L}$, $Q = \omega_0 \frac{L}{R}$

$$\omega_{1,2} = \mp\frac{R}{2L} + \sqrt{\left(\frac{R}{2L}\right)^2 + \frac{1}{LC}}$$

Parallel GCL-> RCL

$\omega_0^2 = \frac{1}{LC}$, $B = \frac{1}{RC}$, $Q = \omega_0 RC$

$$\omega_{1,2} = \mp\frac{1}{2RC} + \sqrt{\left(\frac{1}{2RC}\right)^2 + \frac{1}{LC}}$$

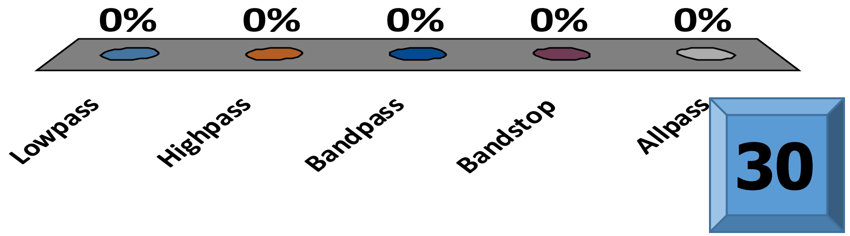


$$H(s) = \frac{s/RC}{s^2 + s/RC + 1/LC}$$

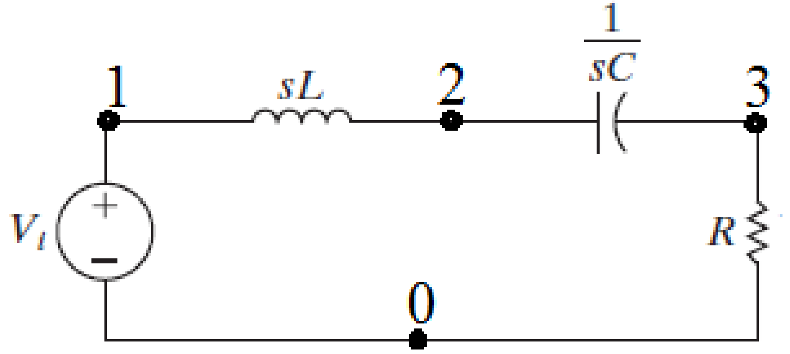
$$K \frac{s^2}{s^2 + B s + \omega_0{}^2}$$

This response is

A. Lowpass
B. Highpass
C. Bandpass
D. Bandstop
E. Allpass



$$\frac{□}{s^2 + Bs + \omega_0{}^2};\ Q = \frac{\omega_0}{B};\ \omega_0^2 = \frac{1}{LC};\ Q = \frac{\omega_0 L}{R}$$



This response can be realized by

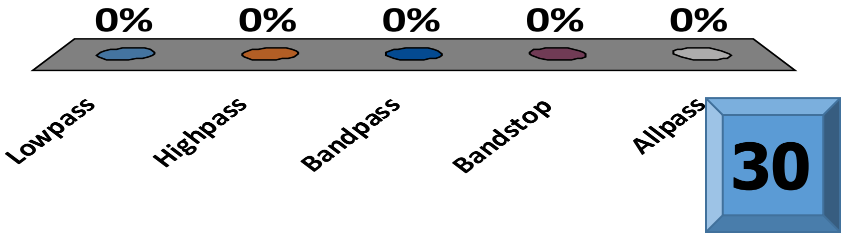
A. V_{21}/V_I

B. V_{31}/V_I

C. V_{32}/V_I

D. V_{20}/V_I

E. V_{30}/V_I

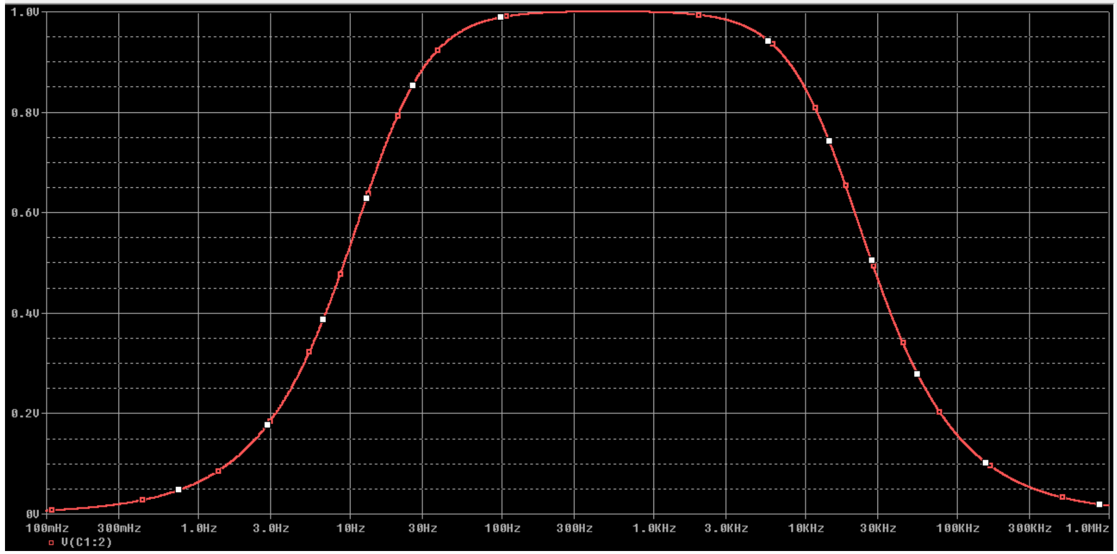


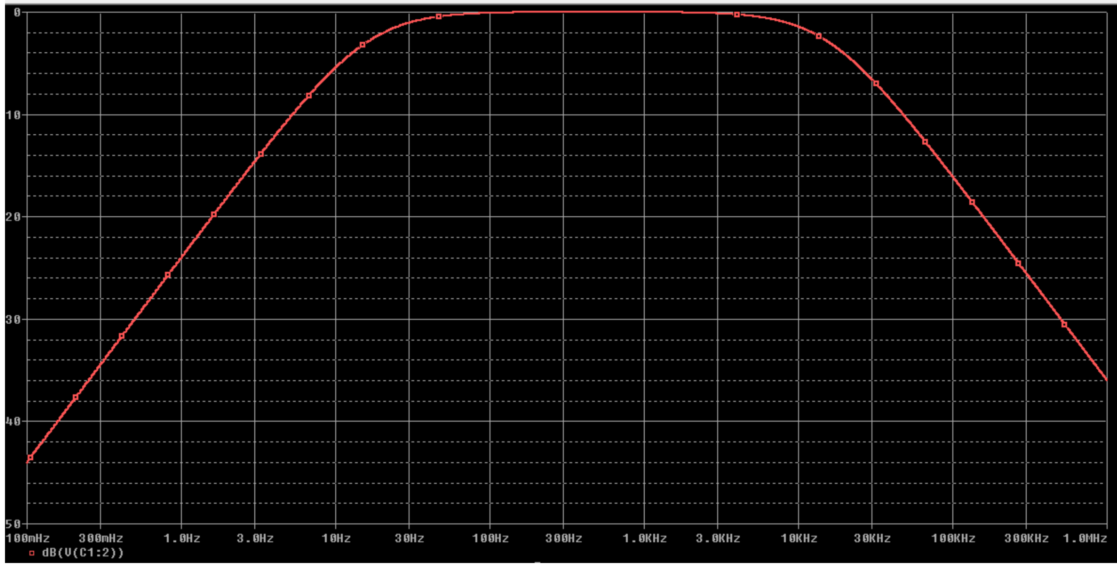
Use of dB's

$$A_{dB} = 20 \log_{10}(\,|\,H(s)|) = 10 \log_{10}(\,|\,H(s)\,|^2),\ \ |\,H\,(s)\,| = 10^{A_{dB}/20}$$

TableForm=

signal ratio	power ratio	dB
0.1	0.01	-20.0
0.141421	0.02	-17.0
0.2	0.04	-14.0
0.3	0.09	-10.5
0.5	0.25	-6.0
0.707107	0.5	-3.0
1.	1.	0.0
1.41421	2.	3.0
2.	4.	6.0
3.	9.	9.5
5.	25.	14.0
7.07107	50.	17.0
10.	100.	20.0

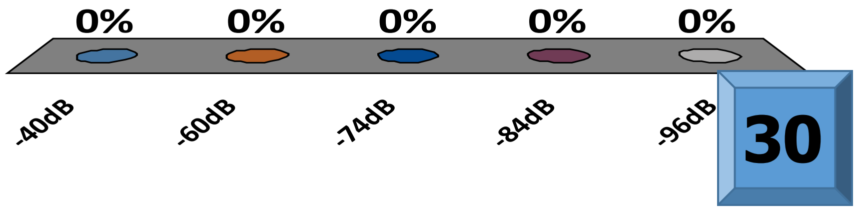




In cascaded circuits signal and power ratio’s are multiplied, dB-values are added.

A signal of 10V is transmitted from an antenna to a mobile phone, where it is amplified with a factor of 500. This amplified signal of 1V is input to the phone's speaker. What is the gain in dB of the signal between the transmitter and the phone?

A. -40dB
B. -60dB
C. -74dB
D. -84dB
E. -96dB



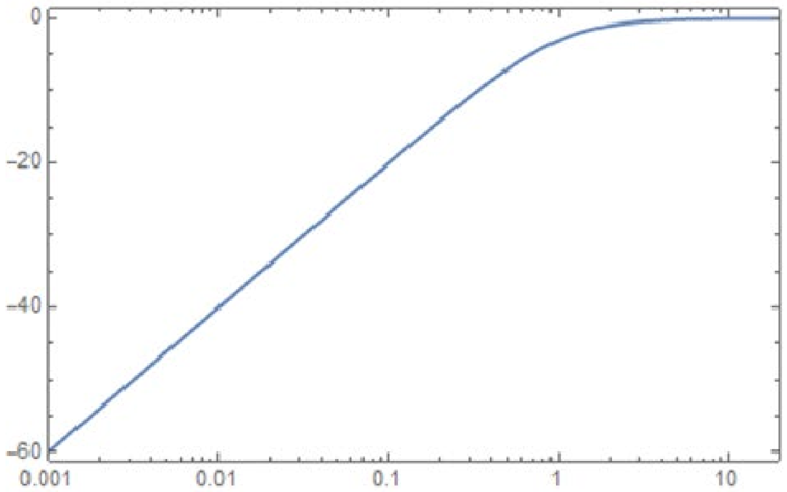
Scaling

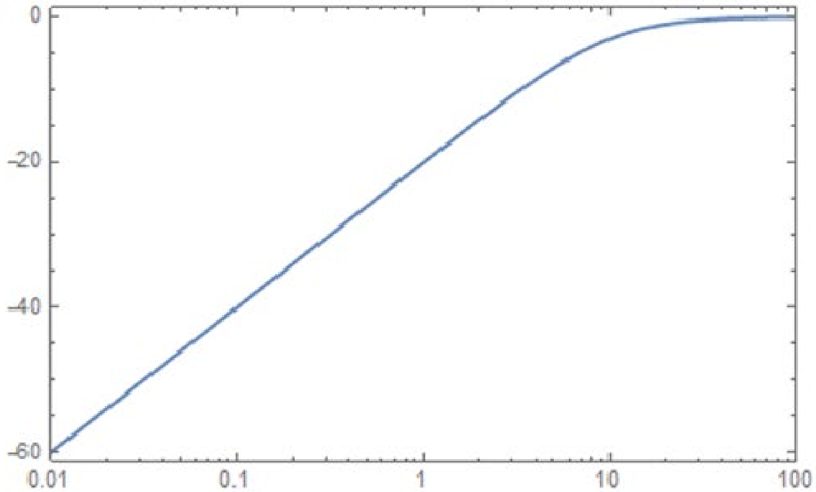
circuit parameter expressions

$R' = k_m R$

$L' = \frac{k_m}{k_f} L$

$C' = \frac{C}{k_m k_f}$





frequency expressions

$\omega' = k_f\, \omega$

$B' = k_f\, B$

$\omega_0' = k_f\, \omega_0$

$Q' = \frac{\omega_0'}{B'} = Q$

$\frac{1}{R' C'} = \frac{k_f}{R C}$

$\frac{R'}{L'} = k_f \frac{R}{L}$

$\frac{1}{\sqrt{L' C'}} = k_f \frac{1}{\sqrt{L C}}$

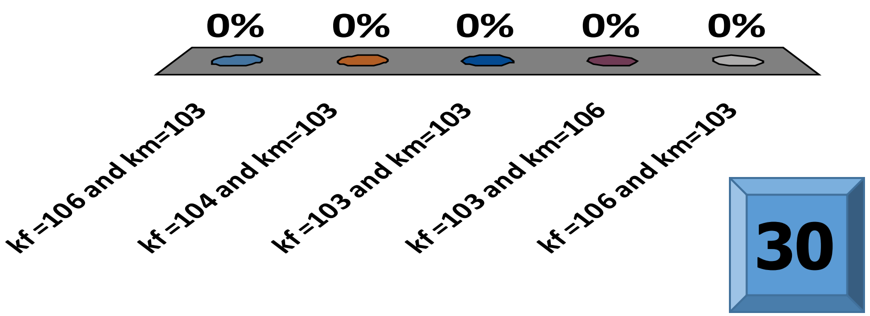
$j\, \omega' L' = j\, k_m\, \omega L$

$\frac{1}{j\, \omega' C'} = \frac{k_m}{j\, \omega C}$

$$R' = k_m R, \qquad \omega' = k_f \omega, \qquad L' = \frac{k_m}{k_f} L, \qquad \text{and} \qquad C' = \frac{1}{k_m k_f} C$$

For the normalized *RL* filter having $\omega_c = 1$ rad/s, $C = 1$ nF, and $R = 1\ \Omega$, it is required to have $\omega_c = 1$ krad/s and $L = 10$ mH. What are the values of k_f and k_m?

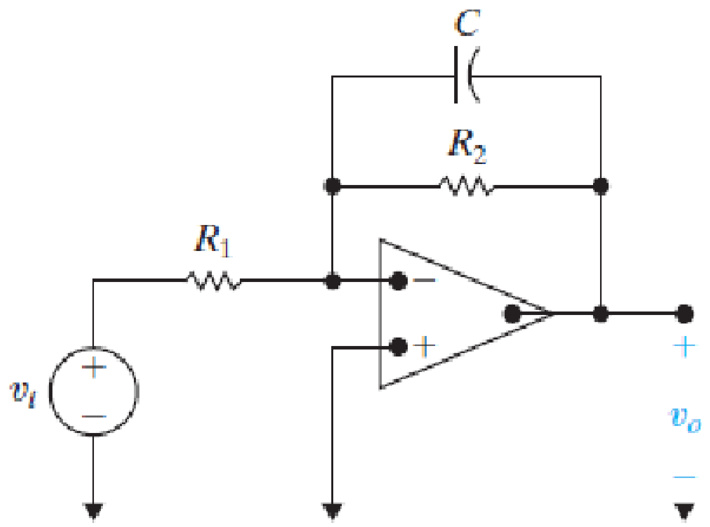
A. $k_f = 10^6$ and $k_m = 10^3$
B. $k_f = 10^4$ and $k_m = 10^3$
C. $k_f = 10^3$ and $k_m = 10^3$
D. $k_f = 10^3$ and $k_m = 10^6$
E. $k_f = 10^6$ and $k_m = 10^3$



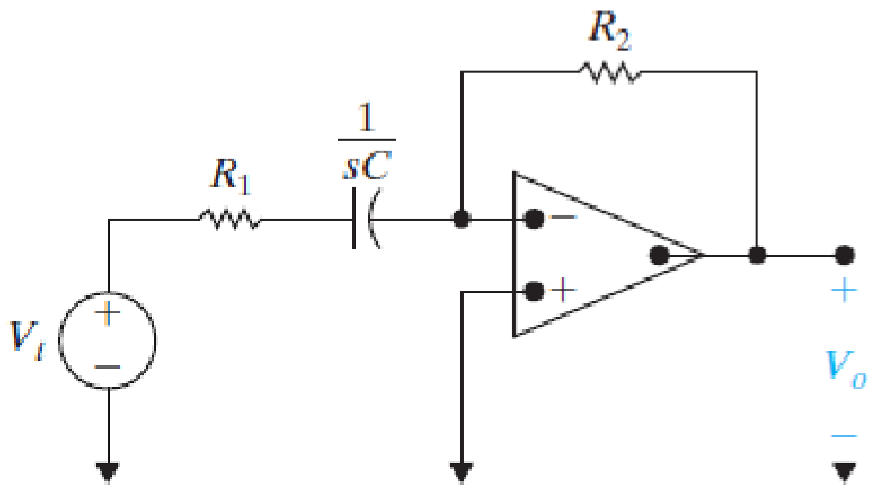
Active filters

- 1st order
  - Low-pass: $K\frac{\omega_c}{s+\omega_c}$

High-pass: $K\frac{s}{s+\omega_c}$



$\omega_c = \frac{1}{R_2 C}, K = \frac{R_2}{R_1}$

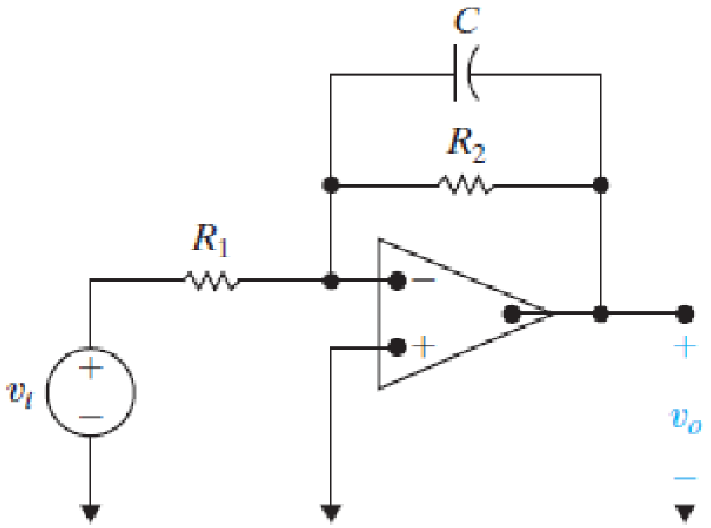


$\omega_c = \frac{1}{R_1 C}, K = \frac{R_2}{R_1}$

$$\omega_c = \frac{1}{R_2 C}, K = \frac{R_2}{R_1}$$

$$\omega_c = \frac{1}{R_1 C}, K = \frac{R_2}{R_1}$$

$C' = 1\mu F$,
$\omega_c' = 10$ krad/s
passband gain =12 dB,
What is value of R_1'?

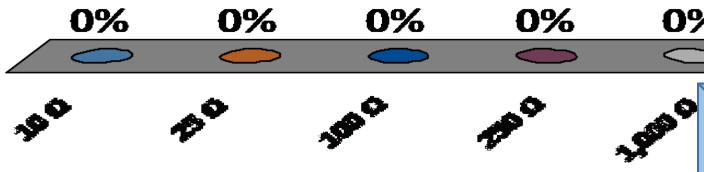


$$R' = k_m R$$

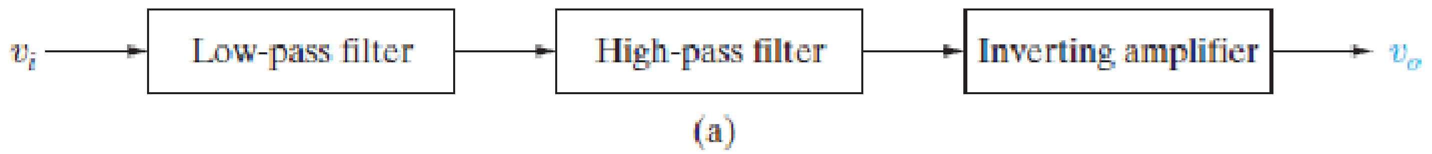
$$L' = \frac{k_m}{k_f} L$$

$$C' = \frac{C}{k_m k_f}$$

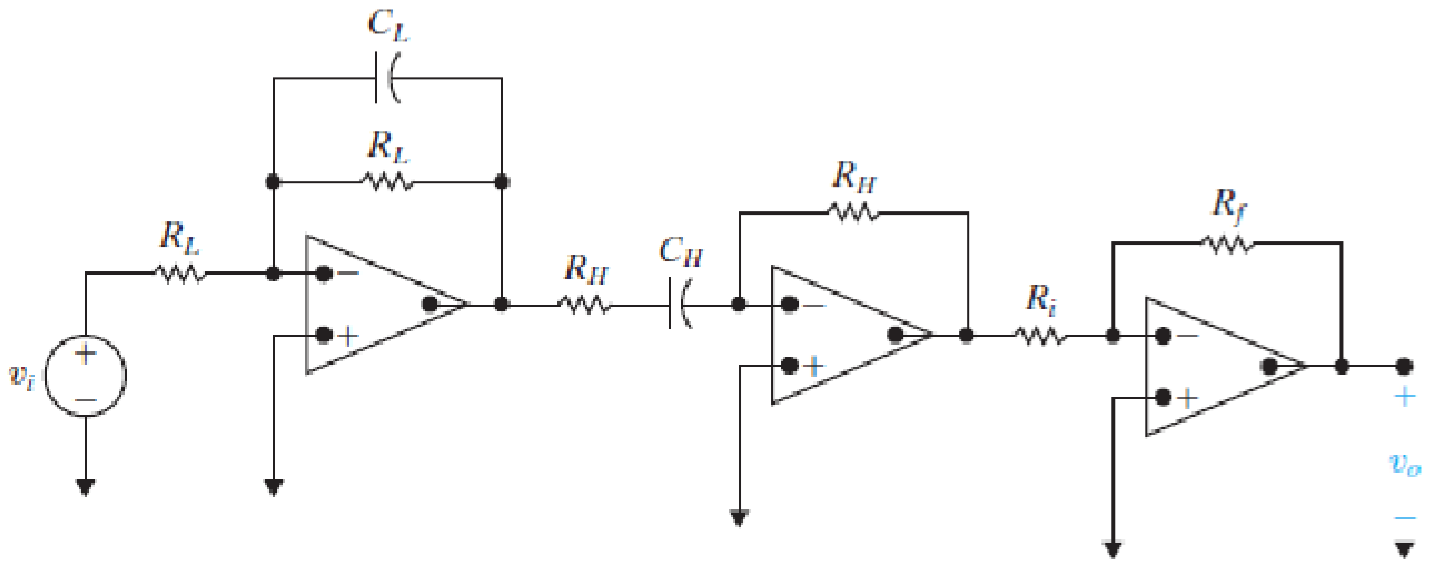
A. 10 Ω
B. 25 Ω
C. 100 Ω
D. 250 Ω
E. 1,000 Ω



Broadband band-pass with cascading LPF and HPF



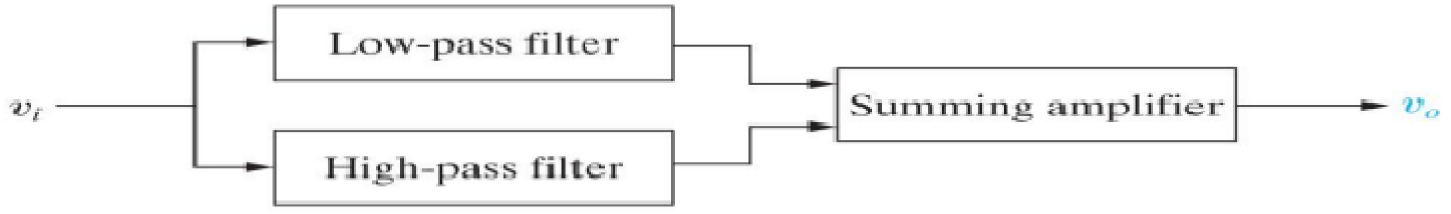
(a)



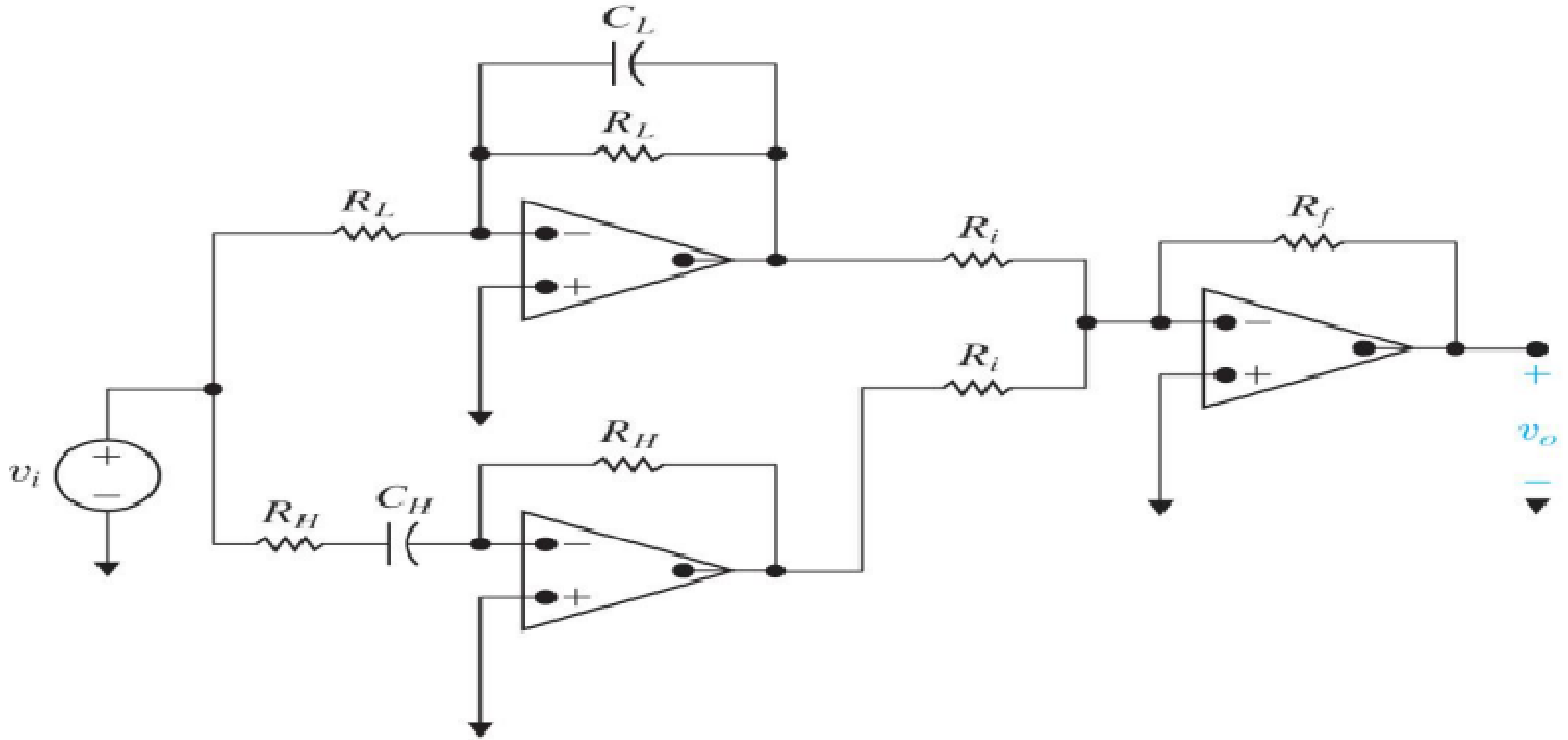
$\omega_{c2} >> \omega_{c1}$

$H(s)=K\frac{Bs}{s^2+Bs+\omega_c^2}$, $\omega_{c1} = \frac{1}{R_H C_H}$, $\omega_{c2} = \frac{1}{R_L C_L}$, $B = \omega_{C1} + \omega_{C2} \approx \omega_{C2}$. $K = -\frac{R_f}{R_i}$

Broadband band-reject with cascading LPF and HPF



(a)



(b)

$\omega_{c2} >> \omega_{c1}$, $\omega_{c1} = \frac{1}{R_L C_L}$, $\omega_{c2} = \frac{1}{R_H C_H}$

passband gain: $\frac{R_f}{R_i}$

Butterworth filters

$$\text{LPF}: H(s) = \frac{1}{B_n(s)}, \ |H(s)| = \frac{1}{\sqrt{1+\left(\frac{\omega}{\omega_c}\right)^{2n}}} = \frac{1}{\sqrt{1+\left(\frac{f}{f_c}\right)^{2n}}}$$

$$\text{HPF}: H(s) = \frac{s^n}{B_n(s)}, \ |H(s)| = \frac{\left(\frac{\omega}{\omega_c}\right)^{n}}{\sqrt{1+\left(\frac{\omega}{\omega_c}\right)^{2n}}} = \frac{\left(\frac{f}{f_c}\right)^{n}}{\sqrt{1+\left(\frac{f}{f_c}\right)^{2n}}}$$

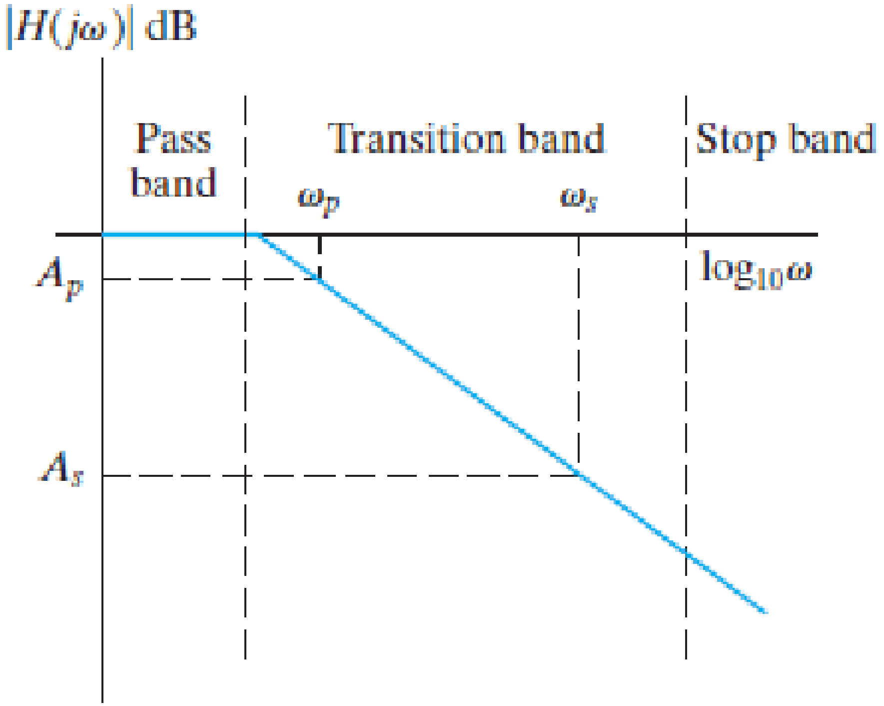


Table 15.2.2 Normalized Butterworth Polynomials of Order *n*

n	**Factors of Polynomial $B_n(s)$**
1	$(s + 1)$
2	$(s^2 + 1.414s + 1)$, where $\sqrt{2}$ = 1.414
3	$(s + 1)(s^2 + s + 1)$
4	$(s^2 + 0.765s + 1)(s^2 + 1.848s + 1)$
5	$(s + 1)(s^2 + 0.618s + 1)(s^2 + 1.618s + 1)$
6	$(s^2 + 0.518s + 1)(s^2 + 1.414s + 1)(s^2 + 1.932s + 1)$
7	$(s + 1)(s^2 + 0.445s + 1)(s^2 + 1.247s + 1)(s^2 + 1.802s + 1)$
8	$(s^2 + 0.390s + 1)(s^2 + 1.111s + 1)(s^2 + 1.663s + 1)(s^2 + 1.962s + 1)$

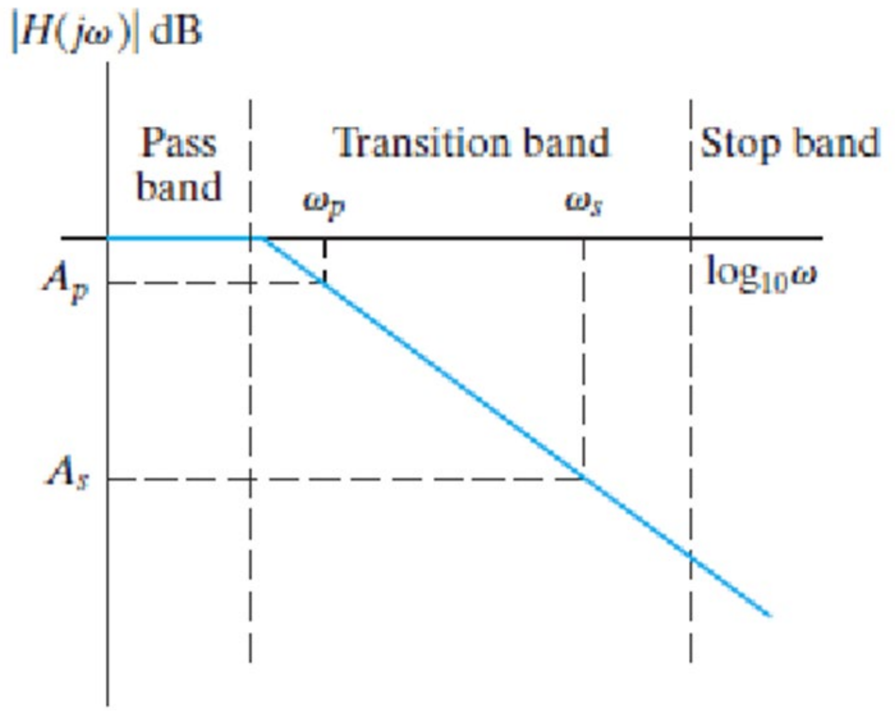
$$\frac{\sqrt{10^{-0.1A_s}-1}}{\sqrt{10^{-0.1A_p}-1}} = \frac{\sigma_s}{\sigma_p},$$

$$n = \frac{\log_{10}\left(\frac{\sigma_s}{\sigma_p}\right)}{\log_{10}\left(\frac{\omega_s}{\omega_p}\right)}, \ \textbf{approximation}: n = \frac{-0.05\,A_s}{\log_{10}\left(\frac{\omega_s}{\omega_p}\right)}$$

For A_s=-20dB at $\omega=2\omega_p$, find required n.

$$\frac{\sqrt{10^{-0.1A_s}-1}}{\sqrt{10^{-0.1A_p}-1}}=\frac{\sigma_s}{\sigma_p},$$

$$n=\frac{\log_{10}\left(\frac{\sigma_s}{\sigma_p}\right)}{\log_{10}\left(\frac{\omega_s}{\omega_p}\right)}, \textbf{approximation : n} = \frac{-0.05\,A_s}{\log_{10}\left(\frac{\omega_s}{\omega_p}\right)}$$



A. 1
B. 2
C. 3
D. 4
E. 5
F. 6